#### NEW DEVELOPMENTS IN THE ANALYSIS OF SHEAR WALL BUILDINGS

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## SYNOPSIS

A new finite element procedure for the static analysis of tall buildings is presented. A common element formulation with linear and bilinear displacement fields is used for the floor slabs and the shear walls. Fictitious beams are included for the transmission of moments in the plane of the elements. The floors are treated as substructures; their stiffness matrix and load vectors are condensed prior to the solution of the system of equilibrium equations. A reduced structure consisting of columns, shear walls and equivalent floor stiffnesses is thus considered. An extension of this work for earthquake analysis is outlined.

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## 1. Introduction

The availability of digital computers and the growing demand for high rise structures have resulted in considerable progress in the analysis of tall buildings during the last decade.

Much work remains to be done, however, regarding their response to dynamic actions such as earthquakes, etc. The problem is further complicated by the existence of torsional modes of vibration that may be induced when the shear centre and the centre of mass fail to coincide.

In order to determine such a response, a stiffness analysis of the system, in which all six degrees of freedom are considered at each node, would be best suited. However, the extremely high number of equations to be solved makes such an approach out of reach of present day computers. It is proposed in this paper that this number can be significantly reduced by recognizing that most tall buildings consist of a restricted number of vertical columns and shear walls connected by a large number of floor slabs of only a few different types, and by treating the latter as substructures.

A computer program for the static, linear and elastic analysis of tall buildings has been developed along these lines using the finite element method. Initial results are presented, and an outline of the work to be done in order to extend its capabilities to dynamic problems is given.

#### 2. Methods of Analysis of Tall Buildings

Conferences [1] and excellent review papers [2] on the analysis of tall buildings have contributed to assess and spread our knowledge of this field and have aroused a great deal of interest.

The duality of the approaches available for the solution of plane elastic shear wall systems subjected to static lateral loads has now been transposed in the analysis of true three-dimensional structures, that is structures that may not be reduced to plane shear wall-frame configurations by virtue of their symmetry. Jaeger et al [3], Rosman [4] and Gluck [5] have extended the continuum method of analysis of such coupled shear walls, and obtain the deflections and internal actions of three-dimensional structures by solving a system of linear differential equations of equilibrium or compatibility [4].

Michael [6] has studied the torsional behaviour of core walls coupled by lintel beams.

Winokur and Gluck [7] and Stamato and Stafford Smith [8], on the contrary, call for the solution of a system of linear equations of equilibrium obtained from a stiffness analysis. Coull and Irwin [9] combine the advantages of both continuum and equilibrium approaches.

In all these cases, coupling between the individual shear walls or frames located in different planes is performed by the floor slabs, which are assumed to be infinitely rigid in their own plane. This assumption is extremely helpful in reducing the number of redundancies considered in the analysis, since only three rigid body degrees of freedom, plus a number of vertical displacements at convenient points are then needed to describe completely the behaviour of the structure under lateral as well as vertical loads. Also,

when evaluating the interaction between these vertical bents the warping rigidity of the floor slabs is neglected. This assumption is not compatible with the usual presence of heavy floor beams perpendicular to the shear walls.

Dickson and Nilson [10] have considered the case of cellular buildings made of continuous shear walls and slabs subjected to membrane actions only.

Goldberg [11] has performed the analysis of structures consisting of parallel shear walls and frames. The warping stiffness of the floor slabs was again neglected; however, the effect of their in-plane deformations was included. This work was extended by Majid and Croxton [12] in order to take into account the axial strains in the load bearing parts of the structure, as well as the effect of eccentric vertical loads. The analysis of a ten-storey building having a length/width ratio of only 3.2 and slab thickness of 3 in. was carried according to this approach. The wind was resisted by plane shear walls situated at the extremities of the building and by interior three bay portal frames. The results were compared to those obtained by assuming rigid floors. A discrepancy of up to 100% was observed in the evaluation of the moments at the base of the central columns [12].

These remarks emphasize the need for a more general method of analysis of building structures in order to describe more accurately even their static behaviour. In the present work, the finite element method is used. All the structural components are represented by an assembly of basic elements connected at nodal points, at which all six physical displacements are considered.

The procedure calls for the following steps:

- Each typical floor is idealized by means of discrete elements; its stiffness matrix and applied load vector are generated.
- 2. Those nodes common to the floor and to the vertical system consisting of columns and shear walls are recognized as boundary nodes; the floor stiffness matrix and load vector are condensed in order that only those degrees of freedom at the boundary nodes be retained.
- Steps 1 and 2 are repeated for all typical floors; the reduced stiffness matrices and load vectors are stored on disc.
- 4. The stiffness matrix and the load vector of the vertical system are generated. At the intersection with each floor, the stiffness coefficients and load vector of the appropriate typical floor are retrieved from storage and added to the above ones.
- The system of equations thus formed for the whole structure is solved, and the displacements at all nodes of the vertical system are obtained.
- When required, the displacements, stresses and strains inside the floors are calculated.

For the sake of the compactness of the program it was decided to adopt the same formulation for all the plate elements, whether they belong to the floor substructures or to the shear walls. Rectangular elements were chosen for their good convergence properties and simplicity. This implies that

structures made of rectangular parallelepipeds, alone, may be analysed by the program described herein. Should the need arise, however, this limitation could be avoided by the introduction of subroutines for the derivation of triangular [13], or even quadrilateral element properties. This would then bring buildings of triangular or curved plan within the capabilities of the program.

With this parallelepiped context, all possible configurations may be accepted: frames, coupled shear walls, enclosed shafts or coupled core walls, slabs, whether monolithic or with holes, etc. The slabs and walls themselves may be reinforced with edge beams, or ribs and diagonal braces may be included. At the time of writing, the development of the program is only in its early stages and the size of the structures that may be analysed is still very small. This point will be discussed at the end of this paper. In the following sections, a detailed description of the element formulations and computation procedures is given, as well as the solutions of a few numerical problems.

# 2. The Finite Element Formulation of the Problem

In order that the finite element method may be used successfully for the analysis of tall buildings, a number of questions inherent in the idealization of the structure must be solved. These are:

- The selection of adequate element displacement functions so that a rapid convergence towards the true behaviour be attained with a minimum number of elements.
- The inclusion of linear beam elements in such a way that a variety of configurations such as rigid frames, diagonal braces, edge beams, lintel beams, etc. be considered within the single idealization pattern.
- 3. The finite element model of plates must take into account the in-plane rotational stiffness of the plate, under unit rotations  $\theta_z$  parallel to the Z axis. (see Figure 1).

# 2.1 The Plate Elements

Considerable efforts have been devoted in the last ten years to the development of finite element formulations. In the solution of tall building problems, however, the list of the available ones is restricted for the following reasons:

- (a) it is advantageous that the degrees of freedom considered at each node be only the physical translations and rotations in the direction of the coordinate axes;
- (b) mid-side nodes should be avoided in order to keep the bandwidth of the stiffness matrices to a minimum;
- (c) the displacement functions should be conforming and the edge displacements of adjacent elements situated in orthogonal planes should be compatible.

A typical element and a set of coordinate axes are shown in Figure 1, with the positive directions of the translations and rotations as indicated. The element properties are developed within the limitations of the small deflection plate theory. For membrane action, the displacements are given by -

in which

$$\{\alpha_{m}\} = \left[\alpha_{1} \quad \alpha_{2} \quad \dots \quad \alpha_{8}\right]^{T} \tag{2}$$

The corresponding stiffness matrix [K] is derived in the classical and well-documented manner [14]. In this formulation, no provision is made for the evaluation of the stiffness of the element under a unit  $\theta$  displacement. This point will be discussed in the next section. For the transverse action, new displacement functions were developed in order that the above requirements be met. They are an extension of the work done previously by Melosh [15], Utku [16] and Mufti [13] on triangular elements. The displacement functions are:

where

$$\left\{ \alpha_{s} \right\} = \left[ \alpha_{9}' \ \alpha_{10}' \ \alpha_{11}' \right]^{\mathsf{T}}$$

$$\left\{ \alpha_{b} \right\} = \left[ \alpha_{13} \ \alpha_{14} \ \ldots \ \alpha_{20} \right]^{\mathsf{T}}$$

$$(5)$$

It can be readily verified that the functions (4) may be determined uniquely within the rectangular element in terms of the rotations  $\theta$ ,  $\theta$  at the nodes. The displacement function (3) is determined within each triangular subregion of the element, in terms of the nodal transverse displacements.

It should be noted that w and  $\theta$  ,  $\theta$  are expressed separately, and that their displacement functions are inconsistent. As is well known, this

normally produces nodal forces under rigid body motions [16]. This difficulty is circumvented in the present case by evaluating separately the contributions of w and  $\theta$ ,  $\theta$  to the stiffness of the element, and by restoring the equilibrium of the nodal forces by means of an equilibrium algorithm.

The details of this procedure have been reported elsewhere [17], wherein very good convergence properties were obtained. Figure 2 shows the convergence graph for the central deflection of a square simply supported plate under uniformly distributed load. The curves obtained for triangular elements with Utku's linear displacement function [16] and Tocher's cubic function [13] have also been plotted. Results of other plate problems under concentrated loads, as well as of cylindrical shells and folded plates have also been reported [17].

# 2.2 Beams and Braces

In addition to the rectangular plate elements described above, linear elements are also included in the program. The six displacements and rotations of Figure 1 are considered at each end of the beam, and these lead to a 12 x 12 stiffness matrix which is readily available in the literature [18]. The organization of the program is as follows:

Each beam element is assigned to a rectangular element, of which it connects two nodes. Conversely, one rectangular plate element may "support" up to six beam elements along its sides and diagonals. These six elements are marked "a" to "f" in Figure 3. Isolated lintel beams, columns or diagonal braces are likewise assigned to a plate element of zero thickness.

The calculations are organised to proceed element by element viz:-

- the stress, strain and stiffness matrices are generated for the plate itself;
- (ii) the stiffness matrix of the beams is calculated, and both are added together.

Should the thickness of the element be zero, or should no beam be assigned to it, then the corresponding part of the calculations is omitted automatically. This approach has two major advantages; firstly, the required computation time is very small; secondly, since in most cases one of the principal planes of the beams is parallel to the plane element, no further definition of the orientation of the members is required. A flow chart describing this particular aspect of the program has been given in [19]. Provision for offset beams is included.

# 2.3 The In-Plane Stiffness of the Plate Elements

In the formulation of the element, only expressions for the  ${\bf u}$  and  ${\bf v}$  inplane displacements are considered. When six degrees of freedom are considered at each node, zero diagonal terms subsist in the stiffness matrix and singularity problems may arise. The authors have indicated elsewhere [17] how this problem could be avoided by including arbitrary small terms at the intersection of those rows and columns of the matrix corresponding to M and  $\theta_{\rm Z}$  respectively.

When a concentrated moment  $M_Z$  is transmitted to the plate, however, these terms do not provide an adequate model for the in-plane rigidity. Majid and Williamson proposed that such a model could be obtained by adding equivalent flexural members along the diagonals of the element [20]. In reference [19], the authors have used a modified procedure for the analysis of plane shear walls. Flexural beams were added along the sides of the element in such a way that the resulting stress pattern was as little affected as possible by these fictitious members.

This point is illustrated in Figure 4 which shows the abutment of a lintel beam in a shear wall, at point A. If the dimensions of the elements are large (as in Figure 4a) a horizontal beam AB will transmit the moment  $\mathbf{M}_{\mathbf{Z}}$  at A in the form of vertical reactions at A and B, and a vertical stress pattern will result in the wall. If, on the contrary, the height of the elements is smaller than the half depth of the beam, the moment at A will be best transmitted by short vertical beams AC and AD, in order that it be resisted by two horizontal forces at C and D.

This procedure has been shown [19] to give a very accurate picture of the stress concentrations in the walls. In both cases, complete fixity of the beam at A will be ensured if a large value of the moment of inertia, of the order of 1000 times that of the lintel beam, is assigned to the fictitious beams.

Michael [21] has shown that elastic deformations in the walls due to the stress concentrations at A were the cause of a significant reduction in the fixity of the beam. Using Michael's conclusions, this may be taken into account by assigning to the fictitious beams a total inertia.

$$I_b = \frac{d^2tI}{18}$$
 (7)

where d is the depth of the lintel beam
 t is the thickness of the wall
 l is the length of the fictitious beam

This formula is applicable to the case of an elastic beam embedded in a wall of the same material. Corresponding formulae could be used for other cases, such as the embedment of a steel beam in a concrete wall. The plane shear wall-frame structure shown in Figure 5 has been analysed with the proposed finite element idealization. On Figure 6 the lateral deflections have been plotted, and are compared with the value obtained by Oakberg and Weaver [22]. Figure 7 shows the shear forces and the moments in the left hand floor beams, at their connection to the shear wall. In this problem the length of the beams and columns has been taken from centre line to centre line, whereas Oakberg and Weaver considered the clear spans only. the lateral deflections obtained are slightly greater than in [22], and so are Shear deformations were included in both cases. the moments in the lintel beams.

## 3. The Computer Program and Substructure Analysis

The computer work described in this section is a development of a finite

element program given in Reference [23] and in which the system of equations is generated in a tridiagonal manner, and is solved by forward elimination and backward substitution.

In order that a large part of the program may be used in this substructure analysis, the same organization was retained for the analysis or the typical floors. Their stiffness matrix and load vectors are thus also generated partition by partition. A prime advantage of this scheme is that most transfers of data between the in-core memory of the computer and the storage devices are performed on entire matrices rather than row by row.

The new program, whose general flow chart is given on the next page, consists of one major loop. For a structure containing n types of floors, this loop is executed n + 1 times. During the first n times, the stiffness matrices and load vectors of the floors are generated and condensed. The resulting matrices are stored on disc by direct access under an identification number corresponding to the type of the floor. During the (n + 1)st execution, the vertical structure consisting of the separate shear walls and frames is treated. Its stiffness matrix is generated, again partition by partition. Each partition corresponds to a full horizontal "slice" of the building; when the "slice" corresponds to a floor level, the equivalent stiffness and load vectors obtained above are retrieved from storage and added. The stiffness matrix and load vectors thus obtained are those of the entire structure, after condensation of the floors. It is imperative that the order in which the nodes are numbered in the floors and the vertical system be consistent.

The solution of the system of equations then proceeds as indicated at the beginning of this section. The stiffness matrix of each partition is inverted after decomposition into

$$[K] = [L] \cdot [D] \cdot [L]^{T}$$
(8)

where,

[L] is a lower triangular matrix of unit diagonal terms

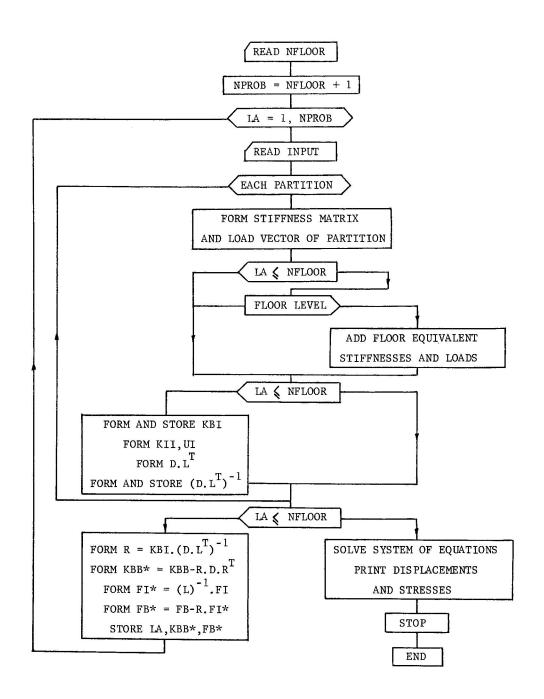
[D] is a diagonal matrix

[L] T is the transpose of [L]

At this stage, only the displacements and stresses in the vertical system are computed. This information is sufficient to calculate the stress resultants in the floor beams, due to the overall displacements of the structure. The displacements, moments and shears within the floor slabs could also be computed without difficulty, at a number of levels specified by the designer. However, the need of such information seems questionable in view of its cost, since the meshsize adopted for the finite element analysis will not necessarily comprise those few locations which are critical for the design of the floor slabs.

In substructure analysis, the stiffness matrix is usually subdivided according to

$$[K] = \begin{bmatrix} KII & KIB \\ KBI & KBB \end{bmatrix}$$
 (9)



where,

KII is the matrix of internal forces due to unit internal displacements,

KBI is the matrix of boundary forces due to unit internal displacements,

KBB is the matrix of boundary forces due to unit boundary displacements.

The load vector is also decomposed into submatrices according to:

$$U = \left\{ \begin{array}{c} UI \\ UB \end{array} \right\} \tag{10}$$

where,

UI includes all the loads applied at interior nodes,

UB includes the loads applied at the boundary nodes.

The computation of the condensed stiffness matrix [KBB $^*$ ] and load vector UB $^*$  requires the inversion of [KII] and the subsequent multiplication [KBI] $\cdot$ [KII $^{-1}$ ] $\cdot$ [KIB]. In our case [KII] may be large and these operations require more memory than is available in the present generation of computers.

An alternative method in which these calculations are performed by decomposition and forward substitution has been proposed by Rosen and Rubinstein [24] and has been used in this work. Some of the calculations are outlined in the flow chart, using the notations of equations 8, 9, 10 and reference [24].

At the time of writing, the maximum size of each partition is only 12 nodes, for a core capacity of 300 Kbytes, or 18 nodes for 400 Kbytes. These figures will be improved as further development work is performed. The number of partitions (which is proportional to the height of the building) is not restricted. Vertical as well as lateral loads may be considered at all the nodes. The core requirement data given above are given for an IBM 360/75 computer, whose maximum capacity is reached. However, the IBM 370 type is already available and should provide much expanded capabilities and improved computation efficiency.

The small structure shown in Figure 8 has been analysed by means of this program. The floor has been idealized by means of nine square elements, whereas the vertical system consists of only two rectangular elements of zero thickness used to define the location of the corner columns. The u displacement at node 5 is given below; it is compared to the values obtained by a simple plane frame analysis performed for two extreme cases of beam stiffness.

program 0.0489
bare frame 0.0606
bare frame + 1/2 width of slab 0.052

For the same problem, Majid and Williamson [20] have obtained 0.0625 for an unspecified Perspex material.

## 4. The Treatment of Dynamic Problems

In the previous section, a means of reducing the size of a building structure for a static finite element analysis has been provided. In the case of earthquakes the response of the structure to the motion of its base consists mainly of horizontal displacements. It is proposed that the procedure described above may also be used for an earthquake analysis.

The additional work to be performed comprises:

- (i) the determination of the consistent mass matrix of the typical floors [25];
- (ii) their condensation along the lines described above;
- (iii) the computation of the consistent mass matrix of the column and shear wall system, to which the equivalent masses of the floors must be added.

At this stage, the size of the condensed structure will in general be too large for an eigenvalue solution and further reduction will be necessary. This will require further condensation of the stiffness and mass matrices and of the load vector. Generally, the number of degrees of freedom will be reduced in such a way that only two horizontal translations per node are retained.

#### 5. Conclusion

In this paper, a new approach to the analysis of tall buildings using finite elements has been presented. A discussion of the major problems that had to be solved has been given, and has been illustrated by means of a few simple examples. In particular, a procedure has been proposed for reducing the size of the problem prior to the solution of the system of equilibrium equations. This technique is being used successfully for the static analysis of elastic structures; the additional steps necessary for an earthquake analysis have been outlined.

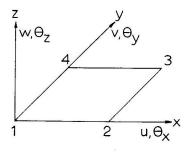
# 6. Acknowledgments

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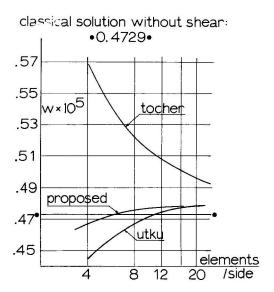
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 $\frac{y}{4}$  c 3 b a a a a a

fig.1. rectangular element and coordinate axes

fig.3. combination of beams and rectangular elements



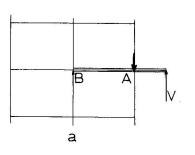


fig.2. simply supported square plate convergence of central deflection

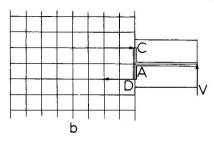


fig.4. two possible models for the embedment of lintel beams

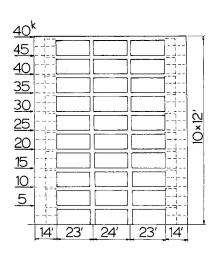


fig.5. idealization of shear wall -frame system

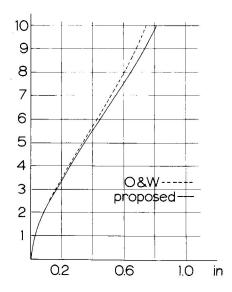


fig.6. shear wall-frame system lateral deflections

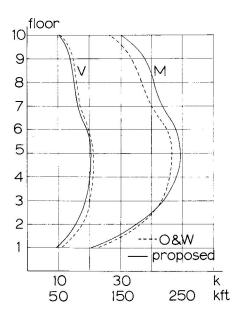


fig.7. shear wall-frame system moments and shears in lintel beams

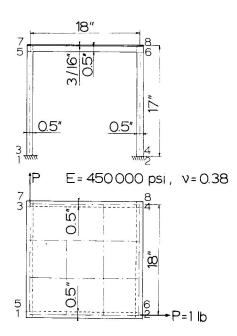


fig.8. table structure

# DISCUSSION OF PAPER NO. 13

NEW DEVELOPMENTS IN THE ANALYSIS OF SHEAR WALL BUILDINGS

by

J.C. Mamet, A.A. Mufti and L.G. Jaeger

### Question by: I. Miller

I wonder if the use of "imaginary beams" to transmit moments into shear walls does not pose some difficulty. If these beams are too stiff, the stiffness matrix will become ill-conditioned. It will be necessary to ensure that the stiffness of these beams is more or less compatible with that of the other structural elements.

Perhaps the addition of rotational degrees of freedom in the element is a safer procedure.

#### Reply by: J.C. Mamet

The authors have experienced no difficulty due to the introduction of fictitious beams for the analysis of shear walls. This may be due to the fact that the flexural properties assigned to these members are always derived from those of actual members. In a coupled shear wall problem, for instance, the moment of inertia chosen ranges between 100 - 1000 times that of the lintel beams. These values derive from convergence studies carried out on simple cases.

Ill-conditioning does occur, however, when one part of the structure is much stiffer than the other, as is the case of shear wall-frame problems (see Fig. 5). On the IBM 360 computer, these problems have been eliminated by resorting to double precision calculations. This of course is independent of the idealization chosen.

As for the addition of rotational degrees of freedom in the formulation of the element properties, the work of Spira and Sokal (26) may be referred to, but the present procedure is certainly simpler and more versatile. Ample information is also given in reference (19) which will appear in the Proceedings of the Institution of Civil Engineers.

(26) Spira, E. and Sokal, Y. Discussion on "New Rectangular Finite Element for Shear Wall Analysis", J.Str. Div. Am.Soc. Civ. Engrs., 1970, (Aug.), 1799-1802.

## Question by: A. Heidebrecht

In the reference which you quoted regarding possible deformation and vibration modes due to in-plane floor deformations, under what conditions were these modes found to occur?

In your single storey example, did you compare the effect of in-plane floor deformations with the case of a floor diaphragm rigid in its own plane? What order of in-plane floor deformations did your calculations show?

## Reply by: J.C. Mamet

The example treated in (27) is a fourteen storey building whose length/width ratio is 12.5. It has a steel skeleton consisting of 18 one bay frames and floor girders on which precast concrete slabs are laid. In order to increase the stiffness of the building in the transverse direction, vertical precast panels are positioned between the columns, thus transforming the 18 frames into continuous shear walls. Concrete is then poured in all the joints and around the exposed steel for fire protection.

The single storey example shown in Fig. 8 was meant to illustrate the procedure used in the program; it has a square slab and therefore no noticeable floor deformation was anticipated. The results indicated are preliminary values; more work is being devoted to that problem. The study of a two storey, two bay structure has also been undertaken.

(27) Adachi, N. et al., "Forced Vibration Test of a Fourteen Storied Prefabricated Apartment House", Annual Report, The Kajima Institute of Construction Technology, Japan, 1969, 18, 461-471.